The Network Representation and the Unloaded Q for a Quasi-Optical Bandpass Filter

MING HUI CHEN, MEMBER, IEEE

Abstract—**A simplified equivalent network for a quasi-optical bandpass filter consisting of several wire-grid polarizers has been developed based upon the concept of local coordinates. Various modes of loss are evaluated, and the equivalent unloaded Q is computed.**

I. Introduction

At millimeter and submillimeter wavelengths, and in the far infrared region, a quasi-optical filter provides lower loss and higher power handling capability than waveguide filters. Saleh introduced an adjustable quasi-optical bandpass filter with design formulas in two recently published articles [1], [2] which demonstrates the practicality of quasi-optical filters. The filter contains several metallic wire grids in space with separations of multiples of a quarter-wavelength. Losses of this filter have not yet been investigated. This paper presents a method for computing the loss characteristic.

The equivalent network presented here is based on a local coordinate concept which leads to a systematic and simplified formulation. As a result, the grating loss [3] can be accounted for in the equivalent network, and the unloaded Q may be evaluated.

II. Modal Representation in Free Space with Rotated Coordinates [4], [5]

Free space may be considered as a uniform waveguide of infinite cross section; therefore, two orthogonally polarized plane waves can be the normal modes in this waveguide. A plane wave propagating along z_0 with any direction of polarization may be represented by the normal modes along $\boldsymbol{x}_0$ and $\boldsymbol{y}_0$ in the coordinate system Σ or $\hat{\boldsymbol{x}}_0$ and $\hat{\boldsymbol{y}}_0$ in the coordinate system $\hat{\Sigma}$ (see Fig. 1). The modal relationships in Σ and $\hat{\Sigma}$ are as follows:

$$\left.\begin{aligned} \boldsymbol{E}(x,y,z) &= \sum_{m=1}^{2} V_m \boldsymbol{e}_m(x,y) e^{-jkz} \\ \boldsymbol{H}(x,y,z) &= \sum_{m=1}^{2} I_m \boldsymbol{h}_m(x,y) e^{-jkz} \end{aligned}\right\} \text{in } \Sigma \qquad (1a)$$

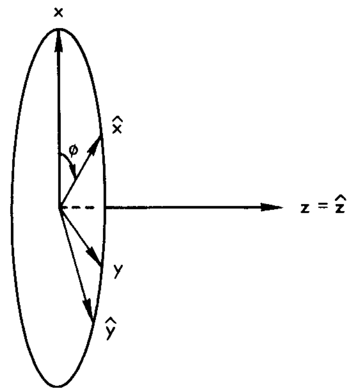

Fig. 1. Geometry of two coordinates Σ and $\hat{\Sigma}$.

$$\left.\begin{aligned} \boldsymbol{E}(x,y,z) &= \sum_{m=1}^{2} \hat{V}_m \hat{\boldsymbol{e}}_m(\hat{x},\hat{y}) e^{-jkz} \\ \boldsymbol{H}(x,y,z) &= \sum_{m=1}^{2} \hat{I}_m \hat{\boldsymbol{h}}_m(\hat{x},\hat{y}) e^{-jkz} \end{aligned}\right\} \text{in } \hat{\Sigma} \qquad (1b)$$

where $\boldsymbol{E}(x,y,z)$ and $\boldsymbol{H}(x,y,z)$ are the electric and magnetic field for the resultant plane waves, V's and I's are the modal voltages and currents that satisfy the transmission line equations, and $\boldsymbol{e}$'s and $\boldsymbol{h}$'s are the modal functions for orthogonal plane waves. Mode functions are defined as follows:

$$\left.\begin{aligned} &\begin{cases} \boldsymbol{e}_1(x,y) = \boldsymbol{x}_0 \\ \boldsymbol{h}_1(x,y) = \boldsymbol{y}_0 \end{cases} \\ &\begin{cases} \boldsymbol{e}_2(x,y) = \boldsymbol{y}_0 \\ \boldsymbol{h}_2(x,y) = -\boldsymbol{x}_0 \end{cases} \end{aligned}\right\} \text{in } \Sigma \qquad (2a)$$

$$\left.\begin{aligned} &\begin{cases} \hat{\boldsymbol{e}}_1(\hat{x},\hat{y}) = \hat{\boldsymbol{x}}_0 \\ \hat{\boldsymbol{h}}_1(\hat{x},\hat{y}) = \hat{\boldsymbol{y}}_0 \end{cases} \\ &\begin{cases} \hat{\boldsymbol{e}}_2(\hat{x},\hat{y}) = \hat{\boldsymbol{y}}_0 \\ \hat{\boldsymbol{h}}_2(\hat{x},\hat{y}) = -\hat{\boldsymbol{x}}_0 \end{cases} \end{aligned}\right\} \text{in } \hat{\Sigma}. \qquad (2b)$$

The above representation yields a relationship between two sets of modal amplitudes which may be written in matrix form as

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The author is with the TRW Defense and Space Systems Group, Redondo Beach, CA 90278.

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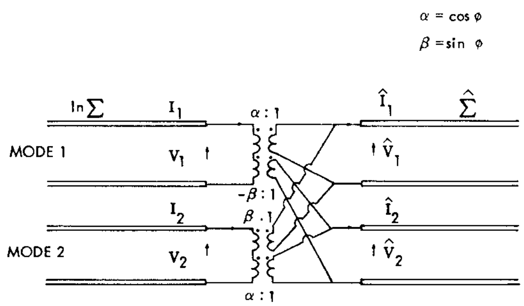


Fig. 2. Equivalent network of mode transformation.

$$\begin{bmatrix} V_1 \\ I_1 \\ V_2 \\ I_2 \end{bmatrix} = R(\phi) \begin{bmatrix} \hat{V}_1 \\ \hat{I}_1 \\ \hat{V}_2 \\ \hat{I}_2 \end{bmatrix} \tag{3}$$

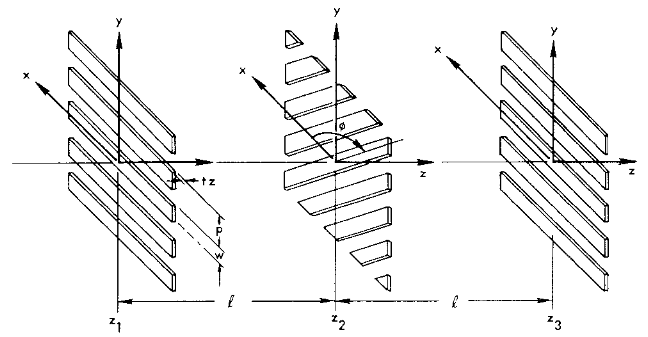
where

$$R(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi & 0 \\ 0 & \cos\phi & 0 & -\sin\phi \\ \sin\phi & 0 & \cos\phi & 0 \\ 0 & \sin\phi & 0 & \cos\phi \end{bmatrix}. \tag{4}$$

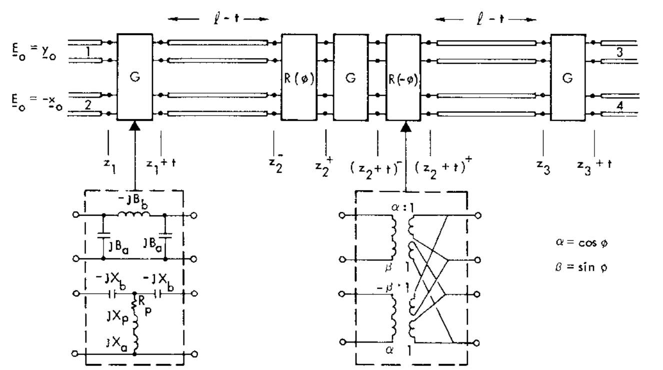
This matrix equation for the modal voltages and currents describes an equivalent network, as shown in Fig. 2, which includes transformers.

III. Network Representation for a Basic Quasi-Optical Filter

A basic quasi-optical filter contains three grids in parallel planes with equal separation as Fig. 3(a) shows [1], [2]. The angular orientation of the middle screen with respect to the first and last screen determines the bandwidth of the filter. This structure exhibits a bandpass response under plane-wave excitation. This is conveniently analyzed by means of an equivalent network. It is well known that, when a plane wave is normally incident upon a parallel wire grating, the grating acts as an equivalent capacitive loading if the incident wave is polarized in the direction perpendicular to the grating; the grating acts as an inductive loading if the incident wave is polarized in the direction parallel to the grating [6]; and a complicated equivalent network results if the incident wave has some other state of polarization with respect to the grating [7]. Therefore, a *complicated* equivalent network would be needed to represent the center screen if a conventional fixed coordinate system were used. However, at each grating, based upon the concept of local coordinates, one can always select a coordinate system whose base vectors x_0 and y_0 are perpendicular and parallel to the grating, respectively. Therefore, the *simple* local coordinate representation of capacitive and inductive loadings can be applied for a grating with any orientation. Consequently, the structure in Fig. 3(a) is represented by the equivalent network in Fig. 3(b). Three identical networks G in Fig. 3(b) represent the three



(a)



(b)

Fig. 3. Geometry and equivalent network for a three-screen filter.

screens with respect to their local coordinates, and the coordinate rotation networks $R(\theta)$ as shown in Fig. 2 are used to connect adjacent local coordinates. In network G, the π and T networks represent the capacitive and inductive loadings, respectively, for a thick grating with loss; these networks would become a shunt capacitance and a shunt inductance if a lossless thin grating were considered. The parameters B_a, B_b, X_a, and X_b may be found in [6]. Since the electric field in the local coordinates is parallel to the grating, an available technique [3] for the computation of loss may be applied. The parameters R_p and X_p are derived, therefore, for loss modification as follows:

$$\frac{R_p}{Z_0} = \frac{X_p}{Z_0} = \frac{1}{\pi r_0}\sqrt{\frac{\omega\mu}{2\sigma}}\ \frac{p}{60\pi} \tag{5}$$

where r_0 is the equivalent radius for the grating strips [6], p is the grating period, ω is the angular frequency, μ is the free space permeability, and σ is the conductivity of the grating strips.

IV. Network Representation for a Multilayer Screen Filter

A multilayer screen filter [1], [2] contains several layers of equally spaced gratings with various orientations. Representing each individual grating in its local coordinate and connecting them with uncoupled modal transmission lines and the coordinate rotation network results in a network representation for the multilayer screen filter as shown in Fig. 4(b).

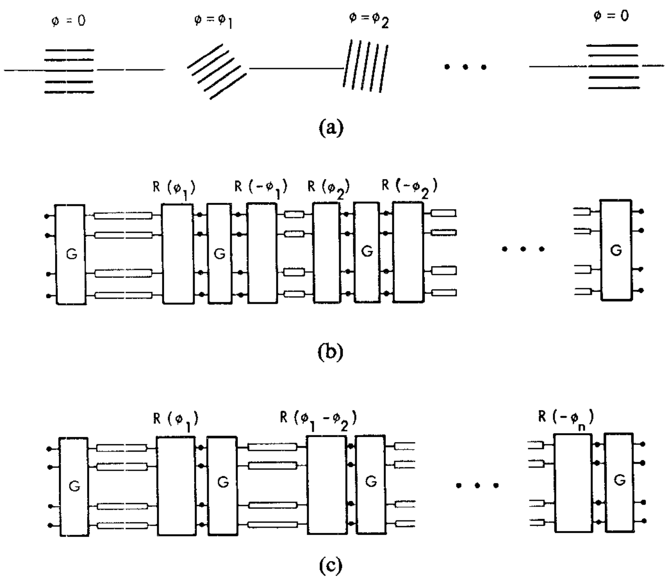


Fig. 4. Geometry and equivalent networks for a multiple-layer screen filter.

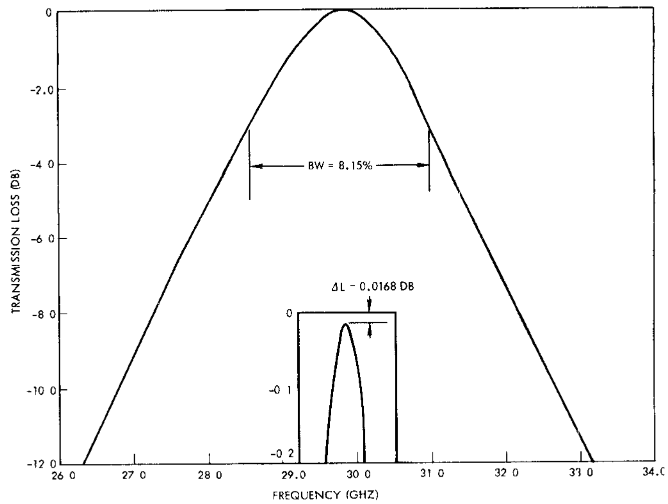


Fig. 5. The transmission characteristics for the $(5/4)\lambda$ mode of a typical three-screen filter of copper wires with $l=12.5$ mm, $p=0.2$ mm, $w=0.12$ mm, and $t=0.01$ mm. The screen-supporting dielectric sheet with thickness of 0.0254 mm, dielectric constant of 2.56, and loss tangent of 0.005 are included.

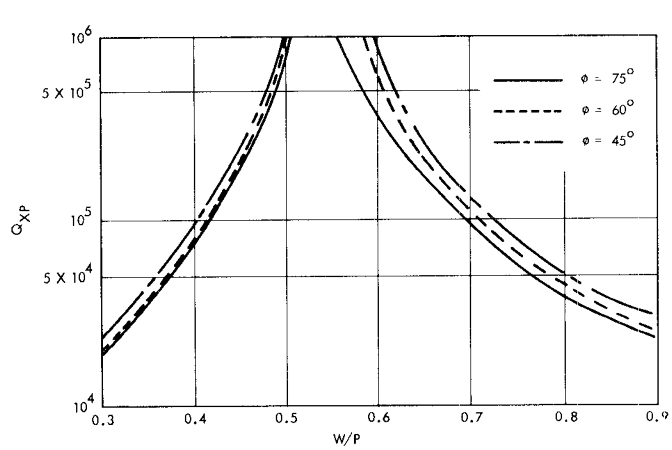


Fig. 6. Q_{xp} characteristics for the $(5/4)\lambda$ mode of a three-screen filter with $l=12.5$ mm, $p=0.2$ mm, $t=0.01$ mm, and $f_0=30$ GHz.

Since the rotation matrix $R(\phi)$ for the coordinate rotation network has the following properties:

$$R^{-1}(\phi)=R(-\phi)$$

$$R(\phi_1)\cdot R(\phi_2)=R(\phi_1+\phi_2) \tag{6}$$

and it commutes with the uncoupled transmission lines, the network in Fig. 4(b) may be simplified as shown in Fig. 4(c).

V. The Loss Characteristics

The insertion loss of this type filter is the sum of the cross-polarization loss, the ohmic loss, and the dielectric loss[1] resulting, respectively, from the energy coupled to the cross-polarized waves, the energy lost on the conducting strips, and the energy dissipated in the dielectric sheets[1] that support the grating strips. These losses may be represented via the conventional way of defining the unloaded Q's as follows:

$$\frac{1}{Q_u}=\frac{1}{Q_{xp}}+\frac{1}{Q_{\text{ohm}}}+\frac{1}{Q_{\text{diel}}} \tag{7}$$

where Q_u is the equivalent unloaded Q related to total loss, and Q_{xp}, Q_{ohm}, and Q_{diel} are the Q's of cross-polarization loss, ohmic loss, and dielectric medium loss, respectively.

The midband insertion loss for a typical filter may be estimated from the 3-dB bandwidth, the unloaded cavity Q, and the number of cavities for the filter [8]. The same equation may also be used to estimate the cavity Q from the given midband insertion loss. Assuming that this three-screen filter is equivalent to a single-cavity filter, the equivalent unloaded cavity Q may be estimated from the computed transmission response for the network in Fig. 3(b). For example, a typical three-screen filter with copper gratings supported by thin dielectric sheets has the computed transmission performance shown in Fig. 5. As is shown in Fig. 5, this typical filter has a midband insertion loss (ΔL) of 0.017 dB and a fractional bandwidth (BW) of 0.0815. The equivalent unloaded Q for this equivalent one-cavity filter is 3170 according to the following equation [8]:

$$Q_u=\frac{4.343}{(\Delta L)_{\text{dB}}\,\text{BW}}. \tag{8}$$

Consequently, Q_{xp}, Q_{ohm}, and Q_{diel} may be separately computed by manipulating the filter parameters in such a way as to have only one type of loss apparent at a time.

Fig. 6 shows Q_{xp} as a function of W/P, the grating width to period ratio, for the grating angles of 45, 60, and 75°. As is shown, the minimum cross-polarization loss corresponds to $W/P\approx0.53$ which might be due to the balance of capacitive and inductive loadings. Q_{ohm} versus

[1]The dielectric sheets were not shown in Fig. 1 for clarity. They are added as support structures for the metal gratings and can be represented by short sections of lossy transmission lines.

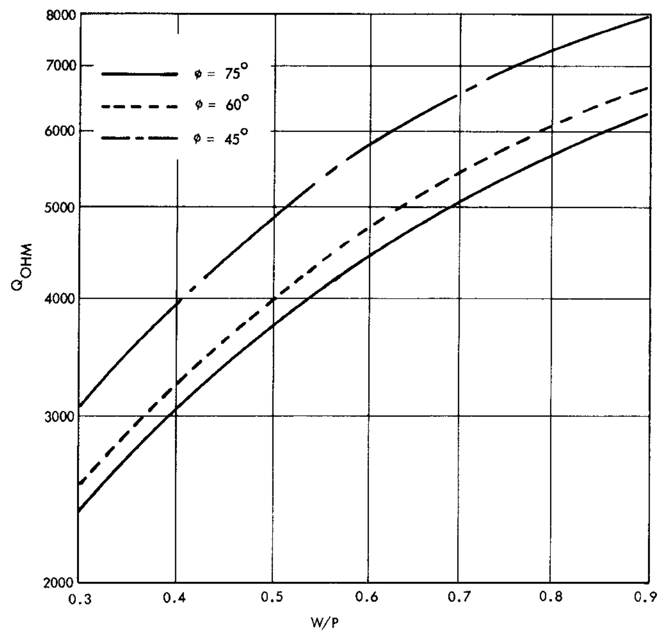


Fig. 7. Q_{ohm} characteristics for the $(5/4)\lambda$ mode of a three-screen filter with $l = 12.5$ mm, $p = 0.2$ mm, $t = 0.01$ mm, $\rho = 1.7241$, and $f_0 = 30$ GHz.

W/P for copper gratings with the angles of 45, 60, and 75° is plotted in Fig. 7. Fig. 8 shows the Q_{diel} characteristics resulting from the fiberglass supporting layers with various loss tangents and thicknesses.

VI. Conclusions

A network representation for structures of multiple layers of gratings has been presented which simplifies the method of analysis and provides better understanding of wave phenomena for this type of filter. Moreover, without this network representation, the available knowledge [3] about the grating loss could not be used in computing the unloaded cavity Q for this type of filter.

Loss characteristics have been analyzed from which a filter with minimum loss can be designed.

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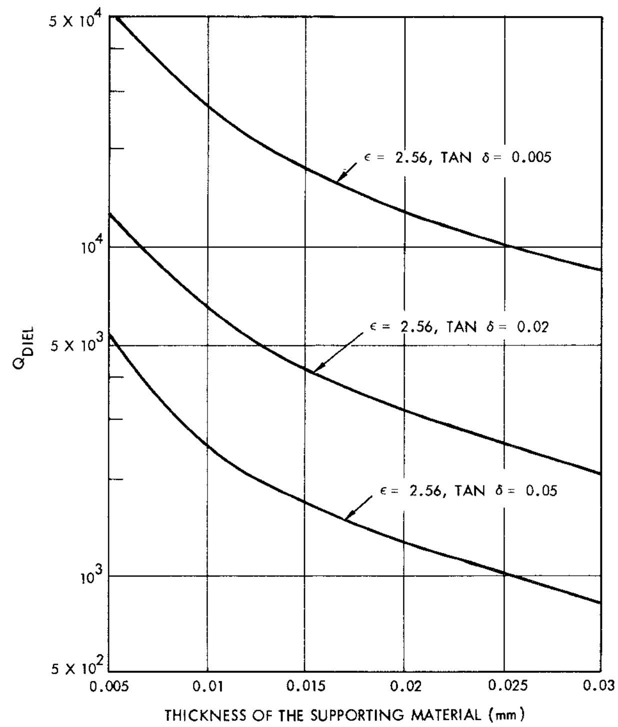

Fig. 8. Q_{diel} characteristics for the $(5/4)\lambda$ mode of a three-screen filter with $l = 12.5$ mm, $p = 0.2$ mm, $t = 0.01$ mm, $\phi = 60°$, and $f_0 = 30$ GHz.

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